LIMITING EXPANSION RATIO OF A SUPERSONIC JET FLOWING ONTO A PERPENDICULAR INFINITE PLANE BARRIER

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The effect of the expansion rate n on the withdrawal Δ of the shock and the relative pressure at the stagnation point is studied for a jet flowing onto an infinite plane barrier placed perpendicular to the jet axis.

It is known [1-3 and others] that one of the principal parameters affecting the nature of the interaction of a supersonic jet with a barrier is the expansion ratio n of the jet, equal to the ratio of the static pressures at the exit cross section of the nozzle P_a and in the surrounding space P_n. When $n < \infty$ a suspended compression shock 1 (Fig. 1) and a nonisentropic compressed layer between this shock and the jet boundary are formed in the jet. The intersection of the suspended shock with the central shock 2, which develops in the jet ahead of the barrier, introduces into the flow behind it disturbances which propagate through the subsonic part of the shock layer in the direction of the jet axis. These disturbances lead to a change in the shape of the central shock and in the parameters behind it in comparison with $n = \infty$.

For a fixed distance h from the nozzle to the barrier the intersection of the shocks 1 and 2 takes place ever closer to the axis with a decrease in the expansion ratio of the jet, and at some value n_* the disturbances introduced by the compressed layer reach the jet axis. A further decrease in the expansion ratio leads to a difference in the amount of axial withdrawal of the central shock and consequently in all the parameters of flow in front of the barrier from their values for $n = \infty$. This expansion ratio n_* is called the limiting value.

Since the flow in the shock layer is determined by the parameters M_a, k, and h, the quantity n_* also depends on these parameters.

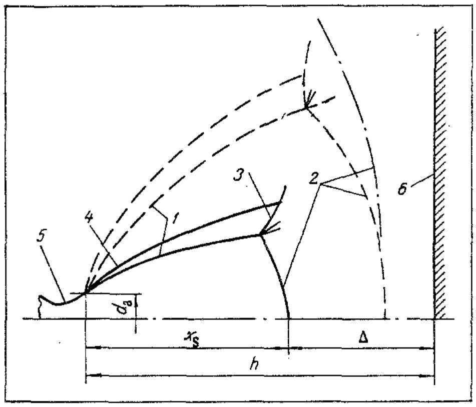

Fig. 1. Diagram of interaction of jet with barrier: 1, 2, 3) suspended, central, and reflected compression shocks; 4) jet boundary; 5) nozzle; 6) barrier; solid, dashed, and dashed-dot lines correspond to jets with expansion ratios $n < n_*$, $n > n_*$, and $n = \infty$, respectively.

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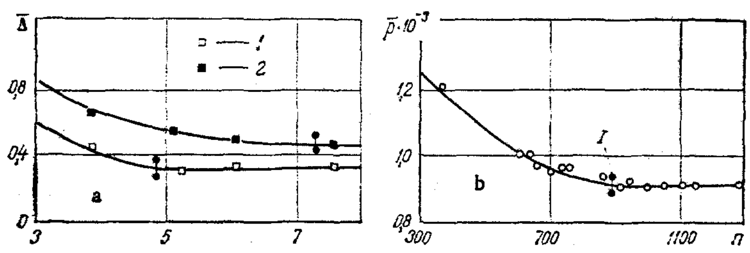

Fig. 2. Withdrawal $\bar{\Delta}$ of central compression shock and pressure $\bar{P}$ at stagnation point as functions of expansion ratio of jet: a) $M_a = 2$, $k = 1.4$; 1) $\bar{h} = 3$; 2) 4; b) $M_a = 2.28$, $k = 1.3$, $\bar{h} = 49.0$; I) adopted value of n_*.

The experimental determination of the limiting expansion ratio was made on the basis of an analysis of the axial withdrawal Δ of the central compression shock and the relative pressure at the stagnation point at the barrier ($\bar{P} = P_b\ P_0$, where P_b is the pressure at the stagnation point and P_0 is the pressure of the adiabatically stagnated stream in front of the central shock).

The tests were conducted on a gas-dynamic stand with an open working section (on jets of cold air) and in a pressure chamber (on jets of CO_2 and Ar) and encompassed the following range of parameters: $M_a = 1$–4, $n = 2$–5000, $k = 1.3$–1.56; in all the modes the barrier did not fall outside the limits of the first barrel of the jet.

The analysis of the schlieren photographs on a discriminator in order to determine the withdrawal Δ was performed with an error of no more than 10%. The accuracy in the determination of the quantity $\bar{P}$ was no worse than 4% on the gasdynamic stand and no worse than 20% in the pressure chamber. In the first case the expansion ratio was varied by changing P_0 and in the second case by the increase in P_n owing to the filling of the pressure chamber with P_0 = const.

A typical graph of the dependence of the withdrawal $\bar{\Delta} = \Delta / r_a$ on the expansion ratio n is presented in Fig. 2a. It is seen that with an increase in n its effect on the withdrawal of the shock continuously decreases, and at some value $n = n_*$ the withdrawal becomes almost constant.

The nature of the dependence of the relative pressure at the stagnation point of the barrier on the expansion ratio is analogous to that of the function Δ(n) (Fig. 2b).

In the measurement of both the withdrawals and the relative pressures the value of the limiting expansion ratio was determined as follows.

From the series of values of $\bar{\Delta}$ (or $\bar{P}$) obtained when clearly $n > n_*$ we calculated the coefficient in the equation of the straight line $\bar{\Delta}$ = const ($\bar{P}$ = const) by the method of least squares and the root-mean-square deviation σ of the experimental points from this straight line. Then the array of points from which the calculation was made was increased by joining to it the values taken at all the smaller expansion ratios. For $n > n_*$ the root-mean-square deviation remains almost constant; for $n < n_*$ its increase begins. The value of the expansion ratio beginning with which a regular increase in σ was determined was taken as n_*.

The values of the limiting expansion ratio found from the withdrawal of the shock and from measurements of the pressures at the center of the barrier are presented in Fig. 3a in the form of a function of the dimensionless argument $h/r_a M_a^2 k$. They are grouped along a straight line with a root-mean-square error of 0.014, and therefore the empirical equation for the determination of n_* has the form

$$n_* = (2.5 \pm 0.2)\frac{\bar{h}^2}{kM_a^2}, \qquad (1)$$

where $\bar{h} = h/r_a$.

Figure 3b illustrates the effect of the discharge parameters on n_*.

By analogy with the limiting expansion ratio one can introduce the concept of the limiting distance h_* to the barrier; when it is exceeded the compressed layer of the jet begins to affect the flow in front of the barrier. From (1) we get

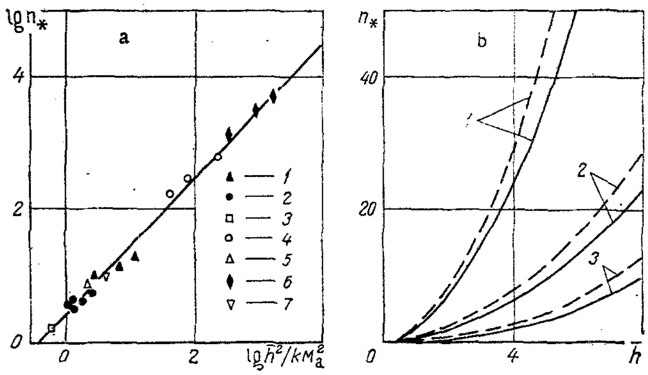

Fig. 3. Dependence of limiting expansion ratio on M_a, k, and the distance $\bar{h} = h/r_a$. The straight line corresponds to Eq. (1). The experimental points are obtained with the following discharge parameters of the jets: a) 1, 2, 3) k = 1.4, M_a = 1, 2, 3 (air); 4, 5) M_a = 2.34, 4.46, k = 1.4; 6, 7) M_a = 2.28, k = 1.3 (CO_2); b) 1, 2, 3) M_a = 1, 2, 3; solid and dashed curves correspond to k = 1.1 and 1.4.

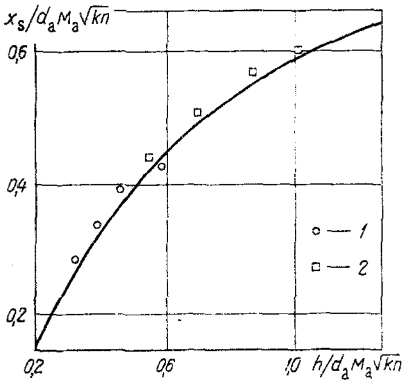

Fig. 4. Comparison of Eq. (3) of [2], obtained with M_a = 1-3, n = 2-40, and k = 1.4, with experimental data; curve: based on Eq. (3); experimental points 1 and 2 correspond to M_a = 2.28, k = 1.3, and n = 4620 and 28.2.

$$\frac{h_*}{r_a} = (0.63 \pm 0.013)\, M_a \sqrt{kn}. \qquad (2)$$

The empirical equations obtained make possible the accurate extrapolation of the results of the calculations and experiments to modes with parameters which differ from the initial parameters. For example, for all expansion ratios $n > n_*$ the withdrawal of the shock from the barrier remains constant, and it can be determined on the basis of numerical calculations of the interaction of a jet with a barrier in a vacuum [1]. For $n < n_*$ the position of the shock in the jet in front of the barrier can be determined from the following empirical equation [2]:

$$\frac{x_s}{d_a M_a \sqrt{kn}} = 0.745 - 0.83 \exp\left(1 - 1.73\, \frac{h}{d_a M_a \sqrt{kn}}\right). \qquad (3)$$

The results of an experimental determination of the position of the central shock in a jet interacting with a barrier in a pressure chamber with $n < n_*$ are presented in Fig. 4. Although the discharge parameters in this case differ sharply from those for which Eq. (3) was obtained, the results of the experiment agree with the calculation from it with good accuracy.

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FLOW OF A STREAM OF UNEVENLY HEATED LIQUID OVER A GAS BUBBLE AT LOW MARANGONI NUMBERS

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The problem of the thermocapillary convection in an unevenly heated liquid near a gas bubble is solved analytically. Estimates are given for the velocity of drift and the shape of the bubble and the vortex boundary.

Let a gas bubble of radius a be placed in a liquid which fills the entire space. A constant temperature gradient $\nabla T = \mathbf{A}$ is maintained at infinity. The force of gravity is absent. Shear stresses producing thermocapillary convection in the liquid develop under these conditions owing to the temperature dependence of the coefficient of surface tension α at the surface of the bubble. The bubble itself begins to move. Under steady conditions the velocity u of this translational motion is constant and is determined in the course of the solution.

The problem can be formulated as a steady-state problem if one changes to a frame of reference connected with the bubble. In such a system the velocity of oncoming flow of the liquid is equal to the drift velocity of the bubble with the opposite sign.

In the report it is assumed that the gas in the bubble is thermally nonconducting and its viscosity is vanishingly small. This allows us not to write the Navier–Stokes equation and the heat-conduction equation for the gas. However, the pressure q in the bubble must be taken into account in writing the boundary conditions.

The problem will be solved in dimensionless quantities. For this we take the following as the characteristic dimensions: the radius a of the undisturbed bubble for the length $|d\alpha/dT|Aa/\eta$ for the velocity, Aa for the temperature, and $|d\alpha/dT|A$ for the pressure. Then the steady distributions of velocities $\mathbf{v}$, pressures p, and temperatures T in the liquid are determined by the system of equations

$$M(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v};\ \nabla\mathbf{v} = 0;\ MP(u + \mathbf{v}\nabla T) = \Delta T. \tag{1}$$

Here all the quantities are dimensionless; $M = |d\alpha/dT|(A a^2/\nu\eta)$ and $P = \nu/\chi$ are the Marangoni and Prandtl numbers. The appearance of the drift velocity u in the heat-conduction equation is connected with the choice of the reference point of the temperature. One can assume that the motion is already established by the starting time. Then it is convenient to measure the temperature from the undisturbed temperature of that point of space at which the bubble is found at the time under consideration upon its continued uniform motion. The partial derivative with respect to time in the nonsteady equation of heat conduction also gives a term proportional to u. The corresponding term of the Navier–Stokes equation vanishes in the chosen frame of reference.

The boundary conditions at the surface of the bubble must be added to the system (1). We take the free surface of the bubble as impermeable and thermally nonconducting, and therefore the normal components of the velocity and heat flux and the normal and tangential components of the stresses vanish at the surface. We write these conditions in a spherical coordinate system r, θ, φ with the polar axis parallel to the vector $\mathbf{A}$.

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